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NONISOTHERMAL FILTRATION OF AN IDEAL GAS

E. A. Bondarev

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Specific features of the statement of boundary-value problems of steady nonisothermal gas filtration are revealed. The solution for an axisymmetric influx to the hole is obtained in quadratures. The influence of nonisothermicity and the process parameters on the pressure distribution is investigated.

Steady nonisothermal gas filtration is described by a system of equations comprised of the continuity and energy equations, the equation of state, and the Darcey law [1]:

$$\begin{gathered} \operatorname{div}(\rho w) = 0; \\ \lambda_{\text{por}} \operatorname{div} \operatorname{grad} T - c_p \rho w (\operatorname{grad} T - \varepsilon \operatorname{grad} p) = 0; \\ w = -\frac{k}{\mu} \operatorname{grad} p, \quad \rho = p/zRT. \end{gathered} \tag{1}$$

For a plane-radial flow, the system (1) can be reduced to the form

$$\frac{dp}{dr} = -\beta Tz/rp; \quad \frac{1}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) - \frac{\nu}{r} \left(\frac{dT}{dr} - \varepsilon \frac{dp}{dr} \right). \tag{2}$$

Here we have introduced the variables and parameters $\bar{p} = p/p_{\text{out}}$, $\bar{T} = T/T_{\text{out}}$, $\bar{r} = r/r_h$, $\nu = c_p M / 2\pi h \lambda_{\text{por}}$, which is an analog of the Peclet number, $\bar{\varepsilon} = p_{\text{out}} \varepsilon / T_{\text{out}}$, and $\beta = \mu M R T_{\text{out}} / 2\pi k h p_{\text{out}}^2$ (here and in the sequel the bar is omitted).

For a unique solution, system (2) requires specification of three boundary conditions. Because it contains the first derivative of pressure and the second derivative of temperature, it would seem necessary to impose one boundary condition for pressure ($p = 1$ at $r = R_c$) and two boundary conditions for temperature. At the same time, it is evident from physical reasoning that the temperature at the bottom of the hole must not be preset arbitrarily during gas extraction (with gas injection to the stratum this difficulty is eliminated). A natural way out of this difficulty is the statement of the second boundary condition for pressure.

For filtration of an ideal gas, system (2) admits an analytic solution. Indeed, at $z = 1$ and $\varepsilon = 0$, it reduces to the form

$$\frac{dp}{dr} = -\beta \frac{T}{rp}; \tag{3}$$

$$\frac{d^2T}{dr^2} + \frac{1-\nu}{r} \cdot \frac{dT}{dr} = 0. \tag{4}$$

We seek the solution to this system under the following boundary values

$$p = 1, \quad T = 1 \quad \text{at} \quad r = R_c; \tag{5}$$

$$p = p_b \quad \text{at} \quad r = 1. \tag{6}$$

The temperature distribution can be obtained from Eq. (4) up to the integration constant in the form

$$T = 1 - \frac{C_1}{\nu} (R_c^\nu - r^\nu) \tag{7}$$

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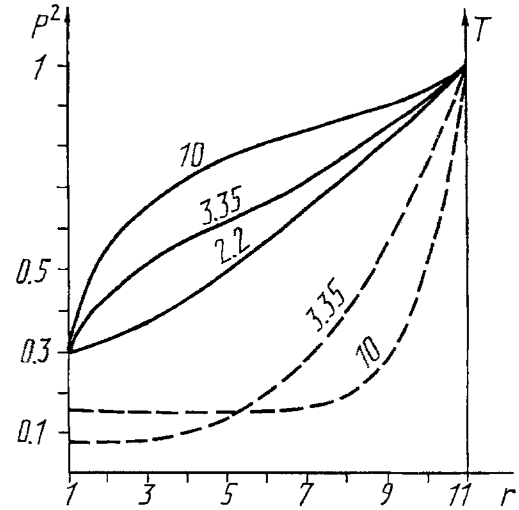

Fig. 1. Distributions of the square of the pressure and the temperature in axisymmetric filtration; numerals at the curves denote values of the dimensionless parameter ν; solid lines denote the square of dimensionless

Substituting Eq. (7) into Eq. (3) and integrating with allowance for conditions (5), we arrive at

$$p^2 = 1 - 2\beta\left[\ln\frac{R_c}{r} - \frac{C_1}{\nu}\left(R_c^\nu \ln\frac{R_c}{r} - \frac{R_c^\nu - r^\nu}{\nu}\right)\right]. \tag{8}$$

Now, using condition (6), we find the integration constant

$$\frac{C_1}{\nu} = \nu\frac{\ln R_c - a}{R_c^\nu(\nu\ln R_c - 1) + 1}, \tag{9}$$

where $a = (1-p_b^2)/2\beta$.

Substituting Eq. (9) into Eq. (7), we obtain the temperature at the bottom of the hole

$$T_b = 1 - \nu\frac{\ln R_c - a}{R_c^\nu(\nu\ln R_c - 1) + 1}(R_c^\nu - 1). \tag{10}$$

It is evident from Eq. (10) that this temperature is substantially dependent on all parameters of the process. Moreover, these parameters must not be prescribed arbitrarily. Using Eq. (10), we may demonstrate that the fulfillment of the condition $T_b \geq 0$ requires that the inequality $a\nu \geq 1$ be satisfied. Moreover, it follows from Eq. (10) that, with increasing parameter ν, the temperature at the extraction point will rise, tending to a limit:

$$\lim_{\nu\to\infty} T_b = a/\ln R_c. \tag{11}$$

Equation (11) is of practical importance, since it allows a determination of the lower limit for gas cooling in filtration through a cylindrical wall based on the input parameters without completely solving the problem of the temperature and pressure distribution.

The pressure distribution in nonisothermal filtration also has its own specific features. In particular, it follows from solution (8) that the curve $p^2(r)$ has an inflection point only provided $\nu a > 1$. The position of this point can be determined from the equation

$$(r/R_c)^\nu = \frac{1}{\nu - 1}\left(\frac{\nu}{C_1 R_c^\nu} - 1\right). \tag{12}$$

The foregoing is illustrated by graphs (Fig. 1) of the pressure distribution for various values of the parameter ν. The calculations are performed for $\beta = 1$, $R_c = 11$, and $p_b = 0.3$; here $a = 0.455$. The temperature at the point of gas extraction was

ν	2.2	3.35	10	∞
T_b	0.007	0.075	0.155	0.190

The predictions comply with the physically obvious conclusion that the degree of cooling of an ideal gas increases with decreasing intensity of its extraction (the parameter ν). Moreover, as this parameter increases, the following trend of temperature variation along the radial coordinate is observed: the gradient on a segment remote from the extraction point increases, after which the curve T(r) goes to a horizontal asymptote (see the dashed curves in Fig. 1).

Bearing in mind that in the case of isothermal filtration

$$\bar{p}^2 = 1 - 2\beta \ln \frac{R_c}{r}$$

and comparing the latter expression with Eq. (8), we find that, for nonisothermal filtration, the pressure is higher than that for constant temperature everywhere, except boundary points.

In conclusion we note that these specific features of formulating boundary-value problems of nonisothermal filtration should also be taken into account in the case where initial system (1) for an ideal gas is reduced to the Laplace equation for the function [2, 3] $u = p^2 + 2bT$, where $b = \lambda_{por} R \mu T_{out}/c_p k p_{out}$.

NOTATION

ρ, gas density; λ_{por}, thermal conductivity of the gas-saturated porous medium; T, gas temperature; c_p, specific heat of the gas at constant pressure; ε, choking coefficient; w, filtration rate; k, permeability of the porous medium; μ, gas viscosity; z, gas imperfection coefficient; R, gas constant; r, radial coordinate; r_h, hole radius; R_c, coordinate of the outer boundary; M, mass flow rate of the gas; h, power (thickness) of the gas-saturated stratum. The subscript out corresponds to the outer boundary of the stratum.

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